

GENERALIZATION OF CONFORMAL TRANSFORMATIONS

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Conformal transformations of a Euclidean (complex) plane have some kind of completeness (sufficiency) for the solution of many mathematical and physical-mathematical problems formulated on this plane. There is no such completeness in the case of Euclidean, pseudo-Euclidean and polynumber spaces of dimension greater than two. In the present paper we show that using the concepts of analogical geometries allows us to generalize conformal transformations not only to the case of Euclidean or pseudo-Euclidean spaces, but also to the case of Finsler spaces, analogous to the spaces of affine connectedness. Examples of such transformations in the case of complex and hypercomplex numbers H_4 are presented. In the general case such transformations form a group of transitions, the elements of which can be viewed as transitions between projective Euclidean geometries of a distinguished class fixed by the choice of metric geometry admitting affine coordinates. The correlation between functions realizing generalized conformal transformations and generalized analytical functions can appear to be productive for the solution of fundamental problems in theoretical and mathematical physics.

Introduction

Conformal transformations play a distinguished role in mathematics and physics. Riemannian and pseudo-Riemannian spaces of constant curvature are not less important (among such spaces are Lobachevsky space and spherical space), their homogeneity is as complete as in the case of a Euclidean space, since their motion groups have the same number of parameters as in the Euclidean case [1]. This work studies only Finslerian spaces admitting an affine coordinate system, so in the case of metric spaces, that is why we consider the length element to be the basic concept, and the concept of angle will be considered secondary. The proposed approach (of course, changed slightly) can be also applied for spaces (geometries) having the length element not defined, but with angles between vectors defined in each point.

If V_n is a Riemannian or a pseudo-Riemannian space with coordinates x^i and a metric tensor $g_{ij}(x)$, then the connection coefficients Γ_{kl}^i in this space are well-known to be defined by the following formula:

$$\Gamma_{kl}^i(g) = \frac{1}{2}g^{im} \left(\frac{\partial g_{mk}}{\partial x^l} + \frac{\partial g_{ml}}{\partial x^k} - \frac{\partial g_{kl}}{\partial x^m} \right). \quad (1)$$

If

$$G_{ij}(x) = \Lambda(x) \cdot g_{ij}(x), \quad (2)$$

where $\Lambda(x) > 0$ is a scalar function defined on coordinates, then

$$\Gamma_{kl}^i(G) = \Gamma_{kl}^i(g) + \frac{1}{2\Lambda} \left(\frac{\partial \Lambda}{\partial x^l} \delta_k^i + \frac{\partial \Lambda}{\partial x^k} \delta_l^i - g^{im} \frac{\partial \Lambda}{\partial x^m} g_{kl} \right). \quad (3)$$

Spaces with metric tensors g_{ij} and G_{ij} are called conformally connected [1].

Since connectivity coefficients are transformed by the following formulas when changing the coordinate system:

$$\frac{\partial x^{i'}}{\partial x^i} \Gamma_{kl}^i = \Gamma_{n'p'}^{i'} \frac{\partial x^{n'}}{\partial x^k} \frac{\partial x^{p'}}{\partial x^l} + \frac{\partial^2 x^{i'}}{\partial x^k \partial x^l}. \quad (4)$$

These are conformal transformations of coordinates, realized by functions f^i in some area $W_n \subset V_n$, where the metric tensor g_{ij} does not depend on the point of the space, and they satisfy the following system of equations:

$$\frac{\partial^2 f^i}{\partial x^k \partial x^l} = \frac{1}{2\Lambda} \left(\frac{\partial \Lambda}{\partial x^l} \delta_k^m + \frac{\partial \Lambda}{\partial x^k} \delta_l^m - g^{mp} \frac{\partial \Lambda}{\partial x^p} g_{kl} \right) \frac{\partial f^i}{\partial x^m}. \quad (5)$$

The convolution of both sides of the equations (5) and the tensor g^{kl} over both indexes gives us the following:

$$g^{kl} \frac{\partial^2 f^i}{\partial x^k \partial x^l} = \frac{2-n}{2\Lambda} g^{kl} \frac{\partial \Lambda}{\partial x^k} \frac{\partial f^i}{\partial x^l}. \quad (6)$$

Thus the functions realizing conformal transformation in Euclidean and pseudo-Euclidean spaces are the solutions of the differential equation (6).

For analytical functions of a complex variable (the first type conformal transformations of the Euclidean plane) and for complex conjugate analytical function of a complex variable (the second type conformal transformations of the Euclidean plane)

$$\Lambda = \left(\frac{\partial f^1}{\partial x^1} \right)^2 + \left(\frac{\partial f^1}{\partial x^2} \right)^2, \quad (7)$$

and the equations (5) are valid in the area of analyticity and simple-connectedness.

Generalization of conformal transformations in Euclidean and pseudo-Euclidean spaces

The concept of analogical geometries was introduced in [2]. It is proposed to call geometries analogical in some areas, if these geometries have same dimensions and if there exists a mapping of one area onto another, under which some set of geodesics (extremals) of one geometry is mapped exactly on some set of geodesics (extremals) of the second geometry. Under certain assumptions the similarity of geometries means that there exist coordinate systems in which the differential equations of geodesics (extremals) coincide.

If in some geometry of affine connectedness we add to the connectivity coefficients the tensor

$$T_{kl}^i = \frac{1}{2} (p_k \delta_l^i + p_l \delta_k^i) + S_{kl}^i, \quad (8)$$

where p_i is an arbitrary covariant field, and S_{kl}^i is an arbitrary tensor field, antisymmetric with respect to the lower two indexes, then the geodesic curves will remain the same [1].

Let functions f^i map an area in a Euclidean or a pseudo-Euclidean space with a metric tensor g_{ij} bijectively onto another area in the same space, and suppose also that these functions satisfy the following system of equations:

$$\frac{\partial^2 f^i}{\partial x^k \partial x^l} = \left[\frac{1}{2}(p_l \delta_k^m + p_k \delta_l^m) - g^{mp} \frac{\partial L}{\partial x^p} g_{kl} \right] \frac{\partial f^i}{\partial x^m}, \quad (9)$$

where p_i is a covariant vector field and L a scalar field. Then this map (a coordinate transformation) will be called elementary generalized conformal.

Notice that in the case of using the additional term (8) with non-zero torsion tensor S_{kl}^i to obtain the formulas (9) instead of a generalization additional conditions appear:

$$S_{kl}^m \frac{\partial f^i}{\partial x^m} = 0, \quad (10)$$

as far as all the other additive terms in both sides of the system (9) are symmetric under the permutations of indexes k and l .

It follows from the definition of elementary generalized conformal transformations of Euclidean and pseudo-Euclidean spaces that these transformations and functions f^i realizing them are closely connected with the concept of projective Euclidean geometries [1].

Thus each function (a component) of an elementary generalized conformal transformation satisfies the following scalar equation:

$$g^{kl} \frac{\partial^2 f^i}{\partial x^k \partial x^l} = g^{kl} \left(p_k - \frac{n}{2} \frac{\partial L}{\partial x^k} \right) \frac{\partial f^i}{\partial x^l}. \quad (11)$$

Though for proper generalized conformal transformations the formula (2) is not valid, we will suppose by definition that

$$\Lambda = \Lambda_0 \cdot \exp(L). \quad (12)$$

In certain sense the scalar field Λ defined this way will be a characteristic for the squared coefficient of the space "stress-strain" under an elementary generalized conformal transformation.

To show the non-triviality of such a generalization let us perform a solution of the system (9):

$$f^i = \frac{x^i}{a + b \cdot g_{kl} x^k x^l}, \quad (13)$$

where a and b — are real numbers and

$$\Lambda = \frac{d}{(a - b \cdot g_{kl} x^k x^l)^2}, \quad (14)$$

where d is a real number.

In the case of the Euclidean (complex) plane (x, y)

$$z = x + iy, \quad F(z) = f^1 + i f^2, \quad (15)$$

the function (13)

$$F(z) = \frac{z}{a + bz\bar{z}} \quad (16)$$

is neither analytical nor complex conjugate analytical when $a \neq 0$ and $b \neq 0$, but it realizes an elementary generalized conformal transformation of the plane. When $a = 0$ this function becomes complex conjugate analytical

$$F(z) = \frac{1}{b\bar{z}}, \quad (17)$$

which corresponds to a conformal map of the second type. When $b = 0$ the function $F(z)$ is analytical,

$$F(z) = \frac{1}{a}z, \quad (18)$$

which corresponds to a conformal map of the first type.

Polynumbers H_4

In the space H_4 the fourth power of the length element written in the basis ψ looks like

$$(ds)^4 = d\xi^1 d\xi^2 d\xi^3 d\xi^4, \quad (19)$$

and a conformally connected geometry will have the length element

$$(ds)^4 = \Xi d\xi^1 d\xi^2 d\xi^3 d\xi^4, \quad (20)$$

where $\Xi > 0$ is a scalar field. This geometry is similar to the geometry of affine connectedness with the connectivity coefficients [2]

$$\Gamma_{kj}^i = \frac{1}{2}(p_k \delta_j^i + p_j \delta_k^i) - p_{kj}^i \frac{1}{\Xi} \frac{\partial \Xi}{\partial \xi^{j-}} + S_{kj}^i, \quad (21)$$

where

$$\psi_k \psi_j = p_{kj}^i \psi_i, \quad p_{kj}^i = \begin{cases} 1, & \text{if } i = j = k, \\ 0, & \text{otherwise,} \end{cases} \quad (22)$$

$p_k, S_{kj}^i = -S_{jk}^i$ are arbitrary tensor fields.

Thus we obtain the system of equations for functions f^i , which realize an elementary generalized conformal transformation in the coordinate space of polynumbers H_4 :

$$\frac{\partial^2 f^i}{\partial \xi^k \partial \xi^l} = \left[\frac{1}{2}(p_l \delta_k^m + p_k \delta_l^m) - p_{kl}^m \frac{\partial L}{\partial \xi^{l-}} \right] \frac{\partial f^i}{\partial \xi^m}, \quad (23)$$

where

$$\Xi = \Xi_0 \cdot \exp(-L). \quad (24)$$

Any function analytical with respect to the variable H_4 realizing a one-to-one correspondence between two ares contained in the coordinate space of polynumbers H_4 satisfies the system (23), and at the same time

$$p_i = 0, \quad \Xi = f^1 f^2 f^3 f^4, \quad L = -\ln |\Xi/\Xi_0|. \quad (25)$$

Functions analytical with respect to the variable H_4 are not the only solutions of the system (23). Another solution of this system is the function

$$f^i = \frac{f_0^i \ln \left| \frac{\xi_-^{i-}}{\xi_0^{i-}} \right|}{a + b \ln \left| \frac{\xi_1^1 \xi_2^2 \xi_3^3 \xi_4^4}{\xi_0^1 \xi_0^2 \xi_0^3 \xi_0^4} \right|}, \quad (26)$$

which becomes analytical with respect to the variable H_4 only when $b = 0$. In the formula (26) a , b , ξ_0^i , f_0^i are constants but, of course, they are not all independent. For the function (26)

$$\Xi = \frac{const}{\xi_1^1 \xi_2^2 \xi_3^3 \xi_4^4}. \quad (27)$$

As far as in the space H_4 the following tensor can be defined

$$q_{ij} = p_{ik}^m p_{mj}^k, \quad (q_{ij}) = diag(1, 1, 1, 1), \quad (28)$$

there also exists a twice contravariant tensor q^{ij} ,

$$(q^{ij}) = diag(1, 1, 1, 1). \quad (29)$$

This is why each component of an elementary generalized conformal transformation of H_4 should satisfy the following scalar equation:

$$q^{kl} \frac{\partial^2 f^i}{\partial \xi^k \partial \xi^l} = q^{kl} \left(p_k - \frac{\partial L}{\partial \xi^k} \right) \frac{\partial f^i}{\partial \xi^l}. \quad (30)$$

Comparing the equations (11), (30) and taking into account the formulas (12) (24), we see that the scalar equation (11), solutions of which are the functions realizing generalized conformal transformations in the four-dimensional Euclidean space, and the scalar equation (30) describing the functions realizing generalized conformal transformation in the space H_4 have the same structure:

$$\delta^{kl} \frac{\partial^2 f^i}{\partial \xi^k \partial \xi^l} = \delta^{kl} \left(p_k \mp 4 \frac{\partial l}{\partial \xi^k} \right) \frac{\partial f^i}{\partial \xi^l}, \quad (31)$$

where the coefficient λ of linear "stress-strain" can be expressed in the terms of a scalar field l for the both four-dimensional Euclidean space and space H_4 with the same formula

$$\lambda = \lambda_0 \exp(l). \quad (32)$$

Notice, however, that we cannot claim that p_k and l are the same in the four-dimensional Euclidean space and in the space H_4 . At the same time it would be very interesting to find such a class of elementary generalized conformal transformations, that for all its elements the covariant field $\left(p_k - 4 \frac{\partial l}{\partial \xi^k} \right)_{E_4} = \left(p_k + 4 \frac{\partial l}{\partial \xi^k} \right)_{H_4}$ would be the same in the four-dimensional Euclidean space and in the space H_4 , i.e. that in both cases the functions f^i would satisfy the same scalar equation not only formally. Linear transforms automatically form a subset of such a class of transformations.

Generalized conformal transformations

The preceding constructions allow us to suppose that the system of equations defining elementary generalized conformal transformations of a metric geometry (at this moment Finsler geometry is developed more than enough for the needs of theoretical and mathematical physics) admitting affine coordinates and for which all its conformally connected spaces are always similar to some geometry of affine connectedness has the following most general view in the affine coordinates:

$$\frac{\partial^2 f^i}{\partial x^k \partial x^l} = \left[\frac{1}{2}(p_l \delta_k^m + p_k \delta_l^m) - \Delta_{kl}^{pm} \frac{\partial L}{\partial x^p} \right] \frac{\partial f^i}{\partial x^m}, \quad (33)$$

where Δ_{kl}^{pm} is a symmetric with respect to the lower indexes number tensor in an affine coordinate system of the initial metric geometry, L and p_k are a scalar and a covariant fields; and for conformal transforms the coefficient λ of linear "stress-strain" is expressed in the terms of the scalar field L with the formula

$$\lambda = \lambda_0 \exp(\pm L/m) \equiv \lambda_0 \exp(l). \quad (34)$$

Here λ_0 is a real number and m is a natural number, equal to the order of the Finsler geometry form, by which the length element is expressed, for instance, for Euclidean and pseudo-Euclidean geometries $m = 2$ and for H_4 -numbers $m = 4$.

It follows from the formulas (33) that any linear non-degenerate transformation is elementary generalized conformal with

$$p_i = 0, \quad L = \text{const}. \quad (35)$$

Though we do hope that for all possible tensors Δ_{kl}^{pm} the concept of Finsler geometry is enough (it is possible that the concept of polynomial geometry [3] might be enough), this conjecture (same as the stronger one) needs a rigorous proof.

For non-degenerate polynumber spaces P_n there always exists a tensor q^{ij} (see (28), (29)), that is why in such spaces elementary generalized conformal transformations satisfy the following scalar equation:

$$q^{kl} \frac{\partial^2 f^i}{\partial x^k \partial x^l} = \left(p_k q^{km} - q^{kl} \Delta_{kl}^{pm} \frac{\partial L}{\partial x^p} \right) \frac{\partial f^i}{\partial x^m}. \quad (36)$$

Elementary generalized conformal transformations (33) do not form a group. But all their products (i.e. consequent executions) together with the inverse ones do form a group, which will be denoted as $G_n(\Delta_{kl}^{pm})$ and called a group of generalized conformal transformations. Products of elementary generalized conformal transformation with the inverse of another one are the solutions of the system

$$\begin{aligned} \frac{\partial^2 f^i}{\partial x^k \partial x^l} = & \left[\frac{1}{2}(p_l \delta_k^m + p_k \delta_l^m) - \Delta_{kl}^{pm} \frac{\partial L}{\partial x^p} \right] \frac{\partial f^i}{\partial x^m} - \\ & - \left[\frac{1}{2}(p'_r \delta_s^i + p'_s \delta_r^i) - \Delta_{sr}^{pi} \frac{\partial L'}{\partial x^p} \right] \frac{\partial f^s}{\partial x^k} \frac{\partial f^r}{\partial x^l}, \end{aligned} \quad (37)$$

where p_l, p'_k, L, L' are some fields, Δ_{sr}^{pi} is the same scalar tensor as in the system of equations (33); and the derivatives $\frac{\partial L}{\partial f^p}$ are meant to be explicitly expressed in terms of partial derivatives by x^i .

Generalized conformal transformations can be viewed as transitions in the uniquely characterized by the tensor Δ_{sr}^{pi} subset (class) of projective Euclidean spaces. Let us emphasize once again that it is enough to investigate elementary generalized conformal transformations, because an arbitrary generalized conformal transformation can be constructed as a product of elementary transformations and inverse to elementary transformations.

Generalized analytical functions

If the initial metric space with a number tensor Δ_{kl}^{pm} corresponding to it is polynumber $P_n \ni X$, then analytical functions realize conformal transformations in the area where the Jacobian of their coordinates is different from zero, and a concept of generalized analytical functions can be introduced in this space [3]. Of course, in this case functions realizing generalized conformal transformations are generalized analytical functions of the given polynumber variable. The following problem seems to be more interesting: find a class $\Upsilon(\Delta_{kl}^{pm}) \ni F(X)$ of generalized analytical functions, each element of which is a solution of the system (37).

Notice that if $F_{(1)}(X), F_{(2)} \in \Upsilon(\Delta_{kl}^{pm})$, then $F_{(1)}(F_{(2)}) \in \Upsilon(\Delta_{kl}^{pm})$. It follows from the group properties of generalized conformal transformations.

A generalized analytical function of a polynumber variable $X \in P_n$,

$$F(X) = f^1(x^1, x^2, \dots, x^n)e_1 + f^2(x^1, x^2, \dots, x^n)e_2 + \dots + f^n(x^1, x^2, \dots, x^n)e_n, \quad (38)$$

$X = x^i e_i$, e_i is a basis, satisfies the correlations

$$\frac{\partial f^i}{\partial x^k} + \gamma_k^i = p_{kj}^i \dot{f}^j, \quad (39)$$

where \dot{f}^j is a generalized derivative, tensor p_{kj}^i is defined by the correlations

$$e_k e_j = p_{kj}^i e_i, \quad (40)$$

and the object γ_k^i should change under transition to another coordinate system according to the following law

$$\gamma_{k'}^{i'} = \frac{\partial x^k}{\partial x^{k'}} \frac{\partial x^{i'}}{\partial x^i} \gamma_k^i - \frac{\partial x^k}{\partial x^{k'}} \frac{\partial^2 x^{i'}}{\partial x^k \partial x^i} f^i. \quad (41)$$

If ε^i are the coefficients of the unit's decomposition in the basis e_i then taking into account the following formula:

$$\varepsilon^k p_{kj}^i = \delta_j^i, \quad (42)$$

from the formula (39) we get

$$\dot{f}^i = \varepsilon^m \frac{\partial f^i}{\partial x^m} + \varepsilon^m \gamma_m^i \quad (43)$$

and an analogue of Cauchy–Riemann correlations:

$$\frac{\partial f^i}{\partial x^k} + \gamma_k^i - p_{kj}^i \left(\varepsilon^m \frac{\partial f^j}{\partial x^m} + \varepsilon^m \gamma_m^j \right) = 0. \quad (44)$$

The conditions of correlations (39) integrability (with respect to the functions f^i) are as follows:

$$\frac{\partial}{\partial x^m} \left(-\gamma_k^i + p_{kj}^i \dot{f}^j \right) = \frac{\partial}{\partial x^k} \left(-\gamma_m^i + p_{mj}^i \dot{f}^j \right). \quad (45)$$

If the polynumbers system P_n is non-degenerate and the generalized derivative is also a generalized analytical function $\{\dot{f}^i, \dot{\gamma}_k^i\}$ then each component f^i formally satisfies the following scalar equation:

$$q^{mk} \dot{\nabla}_m \tilde{\nabla}_k f^i = Q_r^i \ddot{f}^r, \quad (46)$$

where

$$Q_r^i = q^{mk} p_{kj}^i p_{mr}^j. \quad (47)$$

For analytical functions of a complex variable this equation becomes

$$\left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right) f^i = 2 \ddot{f}^i \quad (48)$$

and is identical. Thus the field $(2\ddot{f}^i)$ can be considered as the field of a field source f^i for the operator

$$\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}. \quad (49)$$

Consider a two-dimensional non-homogeneous (with the right-hand side) hyperbolic equation in partial derivatives.

$$\left(\frac{\partial^2}{\partial t^2} - a^2 \frac{\partial^2}{\partial x_s^2} \right) u_s = f_s(t, x_s), \quad (50)$$

where t is time, x_s is the coordinate along the string, $u_s(t, x_s)$ is the amplitude of small lateral oscillations of the string, $\rho f_s dx_s$ is the lateral force acting on an element $(x_s, x_s + dx_s)$ of the string, ρ is the mass density. When changing the variables

$$f^i = u_s, \quad at = x, \quad y = x_s, \quad \frac{1}{a^2} f_s(t, x_s) = 2\ddot{f}^i(x, y) \quad (51)$$

the equations (48) and (50) switch places but the right-hand side of the equation (48) is an analytical function of a complex variable (x, y) , which restricts sufficiently the variety of sources.

Thus if the source function (the right-hand side) of a two-dimensional non-homogeneous hyperbolic equation (a wave equation) written in a special form (48) is an analytical function of a complex variable, then one of the solutions of this equation will be the second antiderivative of the source function divided by two.

Except for the equation (48) each analytical function of a complex variable satisfies the Laplace equation, which can be obtained analogically to how the equation (46) was

obtained, having changed the tensor q^{mk} into the tensor g^{mk} , which is inverse to g_{ij} , the metric tensor of the Euclidean plane:

$$g^{mk} \dot{\nabla}_m \ddot{\nabla}_k f^i = 0 \Rightarrow \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) f^i = 0. \quad (52)$$

Similar equations are valid also for analytical functions of an H_2 -variable,

$$X = x + jy, \quad j^2 = 1, \quad (53)$$

but the elliptic and hyperbolic types of equations switch places:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) f^i = 2\ddot{f}^i, \quad \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right) f^i = 0. \quad (54)$$

So, if the source function (the right-hand side) of a two-dimensional non-homogeneous Laplace equation is an analytical function of an H_2 -variable, then one of the solutions of this equation will be the second antiderivative of the source function divided by two.

Thus when changing $C \leftrightarrow H_2$ not only the wave equation and Laplace equation "switch places", but one of them loses the source (the non-homogeneous right side) and another one gains it. It is quite reasonable now to suppose that such symmetry might take place for polynumbers of dimension greater than two and not only for analytical but also for generalized analytical functions.

The scalar equation (46) for analytical functions of an H_4 -variable in the coordinate system of the ψ -basis (22) becomes

$$\left(\frac{\partial^2}{\partial(\xi^1)^2} + \frac{\partial^2}{\partial(\xi^2)^2} + \frac{\partial^2}{\partial(\xi^3)^2} + \frac{\partial^2}{\partial(\xi^4)^2} \right) f^i = \ddot{f}^i, \quad (55)$$

or in the coordinate system (x^0, x^1, x^2, x^3) of the basis $\{1, j, k, jk\}$ consisting of the unit and three symbol units $j^2 = k^2 = (jk)^2 = 1$:

$$\left. \begin{aligned} \xi^1 &= x^0 + x^1 + x^2 + x^3, & \xi^2 &= x^0 + x^1 - x^2 - x^3, \\ \xi^3 &= x^0 - x^1 + x^2 - x^3, & \xi^4 &= x^0 - x^1 - x^2 + x^3 \end{aligned} \right\} \quad (56)$$

that same equation becomes

$$\left(\frac{\partial^2}{\partial(x^0)^2} + \frac{\partial^2}{\partial(x^1)^2} + \frac{\partial^2}{\partial(x^2)^2} + \frac{\partial^2}{\partial(x^3)^2} \right) f^i = 4\ddot{f}^i. \quad (57)$$

Thus if the source function (the right-hand side) of a four-dimensional non-homogeneous Laplace equation is an analytical function of an H_4 -variable, then one of the solutions of this equation will be the second antiderivative of the source function divided by four.

Notice also that in the equations (48), the first one in (54) and (57) one can take an arbitrary linear combination of the source function's components and not change the coordinates, because the index i is free in both sides, and also use the symmetry (which the corresponding polynumbers do not have) of the scalar operators from the right-hand side to change the coordinates not "shuffling" the components of analytical functions. These circumstances extend in a way the corresponding set of source functions.

Conclusion

In the present paper a generalization of conformal transformations of a metric space is proposed. If we restrict ourselves to considering the spaces admitting affine coordinates then generalized conformal transformations of a given metric space can be considered as the group of transitions between the elements of some class of spaces of constant curvature [1].

If the problem of finding a one-to-one correspondence (modulo a discrete group of transformations) between generalized conformal transformations of the space P_n and generalized analytical functions of the polynumber variable P_n is solved then it is reasonable to hope to build a powerful mathematical instrument for solving mathematical problems and problems of theoretical physics appearing in the spaces P_n .

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